# Sensitivity-guided local identification of a parameter-varying distributed population model

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**Short-term objective :** develop **efficient tools** for predicting population dynamics

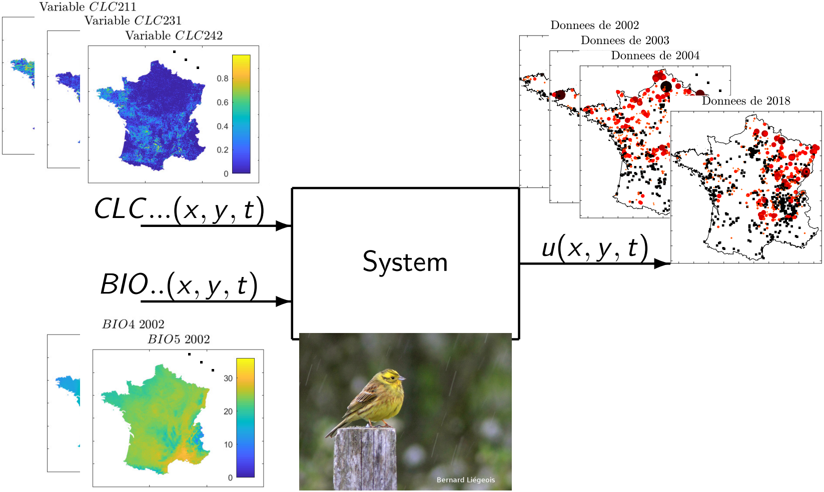
**Medium-term objective :** develop **decision-support tools** to compare scenarios involving climate, agricultural policy, infrastructures, etc.

$\rightarrow$ Models based on **parameter-varying partial differential equations (PDEs)**

# Case study : Yellowhammer

## Data available from the Yellowhammer benchmark

A species affected by agricultural intensification and climate change



Our proposal

→ A **parameter-varying PDE model**

$$\partial_t u = \underbrace{\nabla \cdot (D(H)\nabla u)}_{\text{diffusion}} - \underbrace{\nabla \cdot (W(H,\nabla H)u)}_{\text{advection}} + \underbrace{r(H)u\left(1 - \frac{u}{K(H)}\right)}_{\text{logistic reaction}}.$$

$$\begin{cases} D(H) = D_0\,(1-\alpha H)\,, \\ W(H) = (w_x(1-H), w_y(1-H))\,, \\ r(H) = r_{\max}\,(\beta + (1-\beta)H)\,, \\ K(H) = K_{\max}\,(\gamma + (1-\gamma)H)\,. \end{cases}$$

→ Global changes are included through the environmental heterogeneity $H(t,x,y)$ :

$$\begin{cases} H(t,x,y) \to 1 & \text{for favorable habitat and climate,} \\ H(t,x,y) \to 0 & \text{for unfavorable habitat and climate.} \end{cases}$$

# Problem formulation

- Select the structure of the PDE model.

- Estimate the parameters

$$D_0,\ \alpha,\ w_x,\ w_y,\ r_{\max},\ \beta,\ K_{\max},\ \gamma$$

from population observations

$$\{u(i\,dx, j\,dy, k\,dt)\}_{i=1,\dots,N_x;\ j=1,\dots,N_y;\ k=1,\dots,N_t}$$

**Main difficulties :** small number of time samples, partial observations, noisy data and spatially irregular sampling

**Structure selection :** based on multivariate sensitivity analysis

# Structure selection for the PDE model

## Sensitivity analysis (collaboration with Matieyendou Lamboni – University of Guiana)

Model output, computed with a finite-volume numerical scheme :

$$U = f(D_0, \alpha, w_x, w_y, r_{max}, \beta, K_{max}, \gamma)$$

Input parameters are assumed to be **independent**

**Example for parameter $\alpha$ using the Hoeffding decomposition**

- $f_\alpha(\alpha)$ : **main effect** of $\alpha$ on the output $U$
- $g_\alpha(\alpha, X_{\sim\alpha})$ : **total effect** of $\alpha$, including interactions with the other parameters

# Structure selection for the PDE model

## Sensitivity analysis (collaboration with Matieyendou Lamboni – University of Guiana)

### Sensitivity indices

- $\boldsymbol{GSI_\alpha}$ (first order) : *individual* contribution of $\alpha$ to the variability of $U$
- $\boldsymbol{GSI_{T_\alpha}}$ (total order) : *global* contribution of $\alpha$, including interactions

$$0 \leq \boldsymbol{GSI}_\alpha \leq \boldsymbol{GSI}_{T_\alpha} \leq 1.$$

### Trace-based formulation [1]

Let $M \succeq 0$ be a weighting matrix

$$\boldsymbol{GSI_\alpha} = \frac{\mathrm{Tr}(M\,\mathrm{Cov}[f_\alpha(\alpha)])}{\mathrm{Tr}(M\,\mathrm{Cov}(U))}$$

$$\boldsymbol{GSI_{T_\alpha}} = \frac{\mathrm{Tr}(M\,\mathrm{Cov}[g_\alpha(\alpha, X_{\sim\alpha})])}{\mathrm{Tr}(M\,\mathrm{Cov}(U))}$$

### Numerical implementation

A design of experiments is built by sampling the parameter vector $\boldsymbol{\theta}$ using either Latin Hypercube Sampling (LHS) or Sobol sequences, with $N = 9000$ samples.

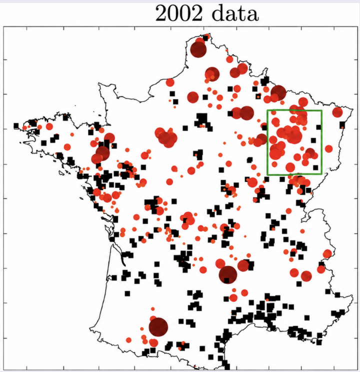
1. Lamboni, M. (2019). *Multivariate sensitivity analysis : Minimum variance unbiased estimators of the first-order and total-effect covariance matrices*. Reliability Engineering & System Safety, 187, 67–92.

Prior knowledge is used to define the following parameter distributions :

### Prior distributions

$$D_0 \sim \mathcal{LN}\left(\ln(137),\, 0.17^2\right),$$
$$w_x \sim \mathcal{LN}\left(\ln(10),\, 1.178^2\right),$$
$$w_y \sim \mathcal{LN}\left(\ln(10),\, 1.17^2\right),$$
$$r_{max} \sim \mathcal{LN}\left(\ln(0.52),\, 0.5^2\right),$$
$$K_{max} \sim \mathcal{LN}\left(\ln(25),\, 0.25^2\right),$$
$$\alpha \sim \mathrm{Beta}(2, 6),$$
$$\beta \sim \mathrm{Beta}(2, 18),$$
$$\gamma \sim \mathrm{Beta}(1, 30).$$

### Study areas



The first-order and total sensitivity indices for the model output in the study area in the following table :

| | $D_0$ | $\alpha$ | $w_x$ | $w_y$ | $r_{max}$ | $\beta$ | $K_{max}$ | $\gamma$ |
|---|---|---|---|---|---|---|---|---|
| $\boldsymbol{GSI_j}$ | 0.014 | 0.447 | 0.406 | 0.008 | 0.040 | 0.002 | 0.009 | 0.0005 |
| $\boldsymbol{GSI_{T_j}}$ | 0.022 | 0.590 | 0.547 | 0.012 | 0.069 | 0.003 | 0.015 | 0.0008 |

The influential parameters are selected using the criterion

$$\boldsymbol{GSI_{T_j}} > \tau, \qquad \text{with } \tau = 1\%.$$

**Influential parameters**

$D_0, \quad \alpha, \quad w_x, \quad w_y, \quad r_{max}, \quad K_{max}$

**Non-influential parameters**

$\beta, \quad \gamma$

# Structure selection for the PDE model

## Reduced model and optimization criterion

### Sensitivity-guided reduced model

For the considered study area :

$$\boldsymbol{\theta}_{\mathrm{red}} = (D_0, \alpha, w_x, w_y, r_{\max}, K_{\max}), \qquad \beta = \widehat{\beta}, \quad \gamma = \widehat{\gamma}.$$

The reduced PDE keeps the same mechanistic form :

$$\partial_t u = \nabla \cdot (D(H)\nabla u) - \nabla \cdot (W(H, \nabla H)u) + r(H)u\left(1 - \frac{u}{K(H)}\right).$$

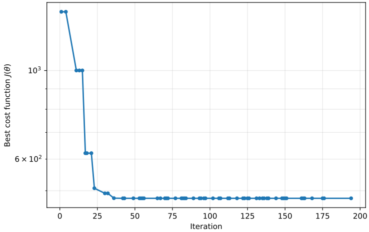
### Local identification

$$\widehat{\boldsymbol{\theta}}_{\mathrm{red}} = \arg \min_{\boldsymbol{\theta}_{\mathrm{red}} \in \Theta_{\mathrm{red}}} \frac{1}{2} \sum_{n} \sum_{i,j} M_{n,i,j} \left(U^{\mathrm{obs}}_{n,i,j} - u(t^n, x_i, y_j; \boldsymbol{\theta}_{\mathrm{red}})\right)^2.$$

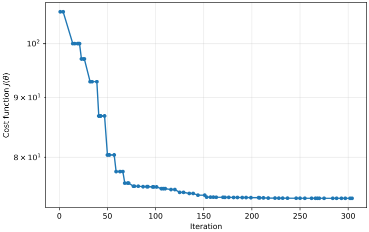
# Results

## Comparison between the full and reduced models

The optimization results for the full and reduced models are shown below.




Reduced model




Full model

The Comparison between the full and reduced models on the study area in the following table :

| Zone | Model | Number of parameters | Final criterion | FIT |
|---|---|---|---|---|
| Study area | Full | 8 | 73.79 | 21.18% |
| Study area | Reduced | 6 | 74.86 | 20.61% |

# Conclusion

- A local identification methodology is proposed for a parameter-varying PDE model.
- Sensitivity analysis is used to select influential parameters and build a reduced model.
- On the study area, the reduced model preserves a fitting quality close to that of the full model, while involving fewer parameters.

## Perspectives

- Extend the procedure to several study areas across France.
- Assess parameter prediction errors using confidence intervals.
- Perform a sensitivity analysis on the criterion $J$ to identify the parameters most sensitive to the available data.

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