Constrained identification methods for robots

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Outline

First part dedicated to description of industrial robots:

- General description;
- Dynamics models;
- Concept of base parameters;
- Usual identification methods…

Second part dedicated to constrained identification applied to robots:

- Choice of constraints;
- Insertion into instrumental variable approach;
- Resolution;
- Experimental validation…

Introduction – General Description of Robots

End effector

n moving links / bodies;
Actuated by DC or synchronous motors;
Considered here as rigid (flexible not a big deal).

Base: immobile.

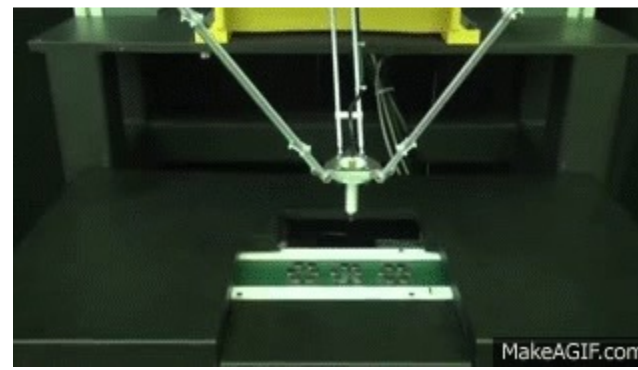
Robots = actuated mechanical structure;
Usually:

- body 1 = shoulder;
- body 2 = arm;
- body 3 = forearm;
- bodies 4 + 5 + 6 = wrist.

Introduction – General Description of Robots

Robots used in manufacturing for pick and place operations ...
... serial manipulators usually enough.

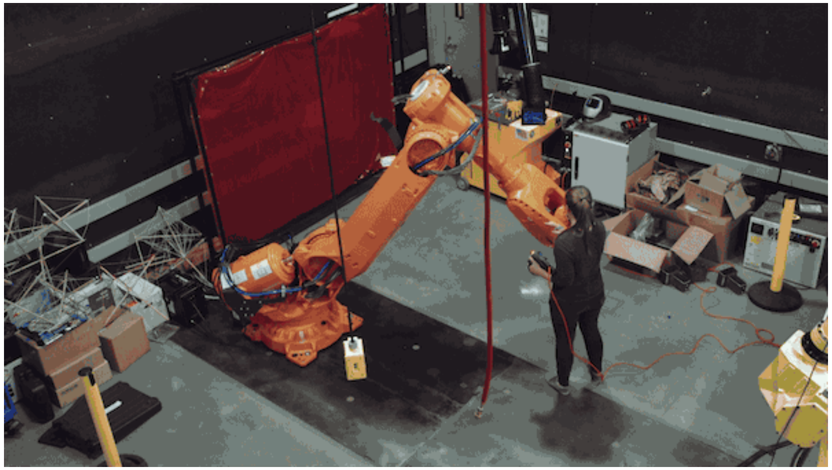
Or used for high-speed positioning...
... parallel robots or XY robots (tables) recommended.

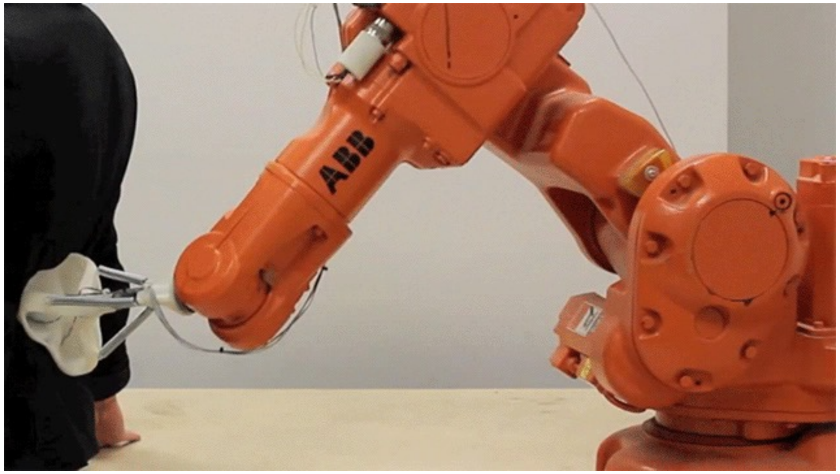


No collaboration/interaction : work in robot mode.

Introduction – General Description of Robots

Now used to collaborate with humans. Serial manipulators, mobile robots…

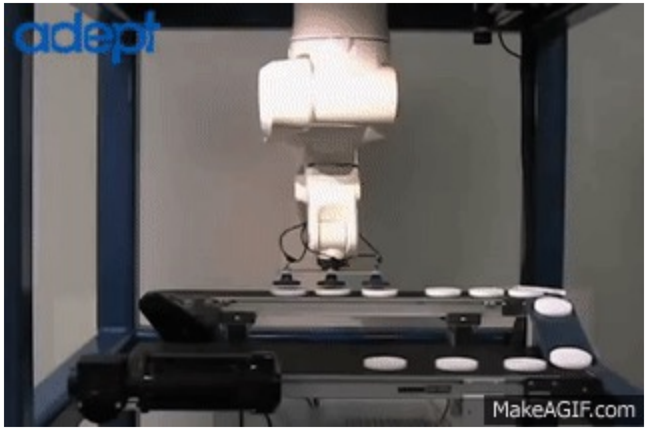




… and even for medical applications! Different structures possible.

Collaboration/interaction : work in collaborative mode (cobot).

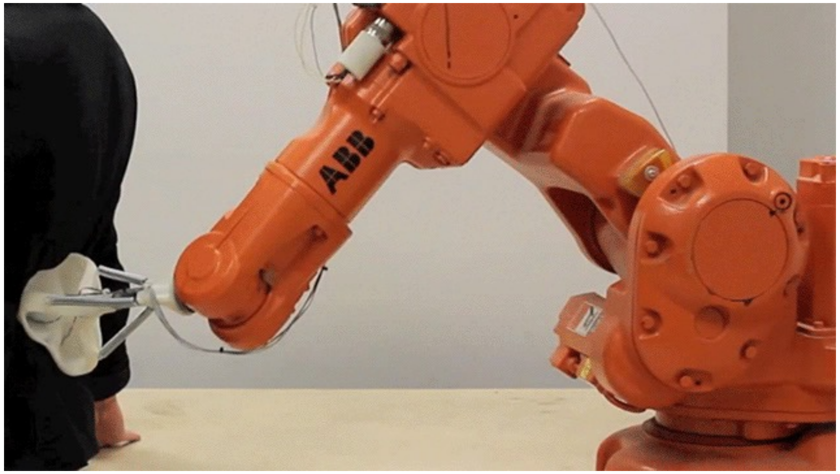
Introduction – Why identifying robots?



Pick-and-place applications:

- Best accuracy and repeatability;
- Movements fast and repeatable.

Only possible with excellent design, mounting, sensing and control!



Collaborative applications:

- Best safety and repeatability;
- Movements slow and repeatable.

Only possible with excellent design, mounting, sensing and control!

Excellent control => excellent knowledge of the dynamic model => identifying it (model based control).

Robot Modeling – Inverse Dynamic Model

(nx1) vector of joint torques/forces

(nx1) vector of joint accelerations

(nx1) vector of joint velocities

(nx1) vector of joint positions

$$\boldsymbol{\tau}_{idm} = \mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{Q}(\mathbf{q}) + \boldsymbol{\tau}_f$$

(nx1) vector of friction torques

(nxn) matrix of Coriolis and centrifugal torques

(nx1) vector of gravity torques

(nxn) inertia matrix of the robot

(nx1) vector that regroups the Coriolis, centrifugal, gravity and friction torques.

$$\mathbf{N}(\mathbf{q},\dot{\mathbf{q}}) = \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{Q}(\mathbf{q}) + \boldsymbol{\tau}_f$$

$$\tau_{f_j} = Fv_j \dot{q}_j + Fc_j sign(\dot{q}_j)$$

Calculated from Newton's laws

viscous friction coefficient

Coulomb friction coefficient

Robot Modeling – Standard Parameters

Inertial parameters of a link j:

$$\boldsymbol{\beta}_{in_j} = \big[\underbrace{XX_j \quad XY_j \quad XZ_j \quad YY_j \quad YZ_j \quad ZZ_j}_{\text{Six components of the inertia matrix}} \quad \underbrace{MX_j \quad MY_j \quad MZ_j}_{\text{Three components of the first moment}} \quad \underbrace{m_j}_{\text{Mass}} \big]$$

Standard dynamic parameters of a link j: $\boldsymbol{\beta}_{inj}$ plus inertia of the actuator I_{aj} and the friction parameters Fv_j and Fc_j:

$$\boldsymbol{\beta}_{STD_j} = \left[\boldsymbol{\beta}_{in_j}^T \quad I_{a_j} \quad Fv_j \quad Fc_j \right]^T$$

Robot with n degrees of freedom: 13n standard dynamic parameters (**usually n = 6** => **78 parameters**)

Robot Modeling – Base Parameters

Inverse dynamic model of robots (IDM) and base parameters:

- Kinetic and potential energies linear/standard parameters;
- So **IDM linear/standard parameters**;
- Standard parameters redundant in the IDM;
- Base parameters calculated by QR factorization or literal expressions.

$$\boldsymbol{\tau}_{idm} = \mathbf{IDM}\left(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}\right)\boldsymbol{\beta}$$

(n x b) matrix of basis functions

(b x 1) vector of base parameters

Base parameters = (structurally) identifiable dynamic parameters.

Direct dynamic model of robots (DDM) :

- joint accelerations in terms of the joint positions, velocities, torques and base parameters;
- State-space written as

$$\dot{\mathbf{x}} = \begin{pmatrix} \dot{\mathbf{q}} \\ -\mathbf{M}^{-1}(\mathbf{q})\mathbf{N}(\mathbf{q},\dot{\mathbf{q}}) \end{pmatrix} + \begin{pmatrix} \mathbf{0} \\ \mathbf{M}^{-1}(\mathbf{q}) \end{pmatrix}\mathbf{u}$$

$\mathbf{x} = \begin{bmatrix} \mathbf{q} \\ \dot{\mathbf{q}} \end{bmatrix}$: (2n x 1) state vector

$\mathbf{u} = \boldsymbol{\tau}_{idm}$: (n x 1) vector of inputs

Robots = **double-integrator systems** => unstable in open loop => **identified in closed loop**.

Robot Identification – Which signals?

$$\tau_{idm} = \mathbf{IDM}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}})\boldsymbol{\beta}$$

τ_{idm}	$\mathbf{q}$	$\dot{\mathbf{q}}, \ddot{\mathbf{q}}$
Accessible via the voltage control	Accessible via encoders/resolvers	Not accessible or rarely measured

Measurements of **q** and **τ**

$\dot{q}, \ddot{q}$ calculated through the differentiation of **q**

Noisy data collected, need of a data filtering to "*clean*" them.

Robot Identification – Data filtering

<u>**Based on the bandwidth of the position closed loop**</u>: $\boldsymbol{\omega_{dyn}}$.

For rigid industrial robots, ω_{dyn} usually around <u>**10-20Hz**</u>.

Two steps run:

Step 1 - Butterworth filter:

- flat amplitude characteristic without phase shift in the range $[0\ \omega_{fq}]$, ω_{fq} being the cutoff frequency;
- in both forward and backward directions to avoid phase distortion;
- appropriate choice: $\boldsymbol{\omega_{fq} \geq 5\ \omega_{dyn}}$.

Robot Identification – Data filtering

Step 2 - Decimate filter:

- reduces the original sampling rate of a sequence to a lower rate;
- lowpass filters the input to guard against aliasing and downsamples the result;
- cutoff frequency given by $\omega_{fp} = 2\pi.0.8.f_m/(2.n_d)$;
- appropriate choice: $\boldsymbol{\omega_{fp} \geq 2\ \omega_{dyn}}$.

The velocities and accelerations, $\hat{\dot{\mathbf{q}}}, \hat{\ddot{\mathbf{q}}}$, calculated with central differentiation of $\hat{\mathbf{q}}$, the filtered positions.

Robot Identification – Base Parameters Estimation

After the data filtering and decimation process, the following over-determined system called IDIM (Inverse Dynamic Identification Model) obtained

$$\mathbf{y}(\boldsymbol{\tau}) = \mathbf{X}\left(\hat{\mathbf{q}}, \hat{\dot{\mathbf{q}}}, \hat{\ddot{\mathbf{q}}}\right)\boldsymbol{\beta} + \boldsymbol{\varepsilon}$$

(r x 1) measurements vector built from the actual torques $\boldsymbol{\tau}$

(r x 1) vector of error terms

(r x b) observation matrix built from $\left(\hat{\mathbf{q}}, \hat{\dot{\mathbf{q}}}, \hat{\ddot{\mathbf{q}}}\right)$

Covariance matrix of $\boldsymbol{\varepsilon}$ chosen as:

(n_e x n_e) identity matrix

$$\boldsymbol{\Omega} = diag\left(\sigma_1^2 \mathbf{I}_{n_e} \quad \cdots \quad \sigma_j^2 \mathbf{I}_{n_e} \quad \cdots \quad \sigma_n^2 \mathbf{I}_{n_e}\right)$$

error variance calculated from OLS solution of the subsystem j

Robot Identification – Base Parameters Estimation

-> The simplest solution: Least-Squares (LS) estimates (**IDIM-LS method**)

$$\hat{\boldsymbol{\beta}}_{LS} = \left(\mathbf{X}^T\mathbf{X}\right)^{-1}\mathbf{X}^T\mathbf{y} \qquad \hat{\boldsymbol{\Sigma}}_{LS} = \left(\mathbf{X}^T\boldsymbol{\Omega}^{-1}\mathbf{X}\right)^{-1}$$

Because robots are identified in closed-loop, LS estimates could be biased, even with a careful data-filtering.

-> Instrumental variable (IV) method deals with that problem (**IDIM-IV method**)

$$\hat{\boldsymbol{\beta}}_{IV} = \left(\mathbf{Z}^T\mathbf{X}\right)^{-1}\mathbf{Z}^T\mathbf{y} \qquad \hat{\boldsymbol{\Sigma}}_{IV} = \left(\mathbf{Z}^T\boldsymbol{\Omega}^{-1}\mathbf{Z}\right)^{-1}$$

IV matrix $\mathbf{Z}$ constructed with the IDM and simulated data resulting from the simulation of the DDM

$$\mathbf{Z} = \mathbf{X}\left(\mathbf{q}_S, \dot{\mathbf{q}}_S, \ddot{\mathbf{q}}_S\right)$$

Robot Identification – Base Parameters Estimation

-> A closed-loop input error (**CLIE**) can also deal with this problem. In that case, one has

$$\hat{\boldsymbol{\beta}}_{CLIE} = \left(\mathbf{Z}^T\mathbf{Z}\right)^{-1}\mathbf{Z}^T\mathbf{y} \quad \hat{\boldsymbol{\Sigma}}_{CLIE} = \left(\mathbf{Z}^T\boldsymbol{\Omega}^{-1}\mathbf{Z}\right)^{-1}$$

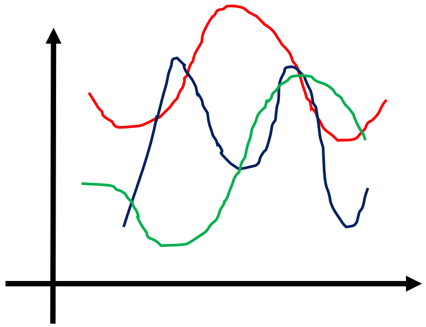
Matrix $\mathbf{Z}$ still constructed with the IDM and simulated data resulting from the simulation of the DDM

$$\mathbf{Z} = \mathbf{X}\left(\mathbf{q}_S, \dot{\mathbf{q}}_S, \ddot{\mathbf{q}}_S\right)$$

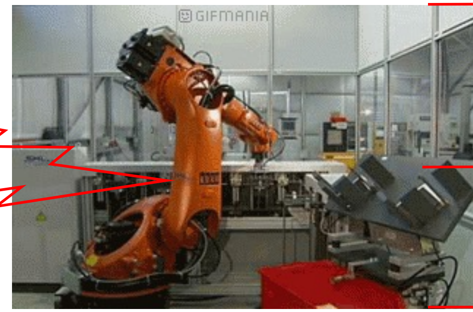
-> Note that IV and CLIE are iterative algorithms (usually converge in 5 iterations – very fast). The usual closed-loop output error not recommended/appropriate for robot identification: lack of sensitivity.

-> Note also that with CLIE or IV we validate the IDM and DDM.

Robot Identification – In one slide



Exciting trajectories



Collecting data

τ_1 , q_1,

…

τ_i , q_i,

…

τ_r , q_r.

Data filtering to get $\hat{\mathbf{q}},\hat{\dot{\mathbf{q}}},\hat{\ddot{\mathbf{q}}}$ Then build $\mathbf{y}(\boldsymbol{\tau}) = \mathbf{X}\left(\hat{\mathbf{q}},\hat{\dot{\mathbf{q}}},\hat{\ddot{\mathbf{q}}}\right)\boldsymbol{\beta} + \boldsymbol{\varepsilon}$

Finally estimate $\boldsymbol{\beta}$ with the method of your choice.

☺ Robot identification is easy ☺

Robot Identification – Timeline

2002: Method based on IDM and Maximum Likelihood introduced by Olssen and Swevers

2013-2014: A new Input-Error method and a method based on IDM and IV introduced by Gautier and Janot

1986: Method based on IDM and LS introduced by Gautier

1980 | 1990 | 2000 | 2010 | 2020

Efforts made on robot modeling. No systematic Identification procedure.

Method validated on several industrial robots and prototypes. Extension to other systems (compactors, cars, tires…)

More complex methods now considered thanks to the increase of computational power

Robot Constrained Identification – Constraints

Until now, we have dealt with unconstrained identification.
Theoretical and experimental results showed it was enough.
In 2014, Courtesao and Sousa proposed to use the following constraints to "*improve*" the LS estimates:

$$\mathbf{J}\left(\boldsymbol{\beta}_{STD_j}\right) = \begin{bmatrix} \frac{1}{2} tr\left(\mathbf{I}_j\right)\mathbf{I}_3 - \mathbf{I}_j & \mathbf{MS}_j \\ \mathbf{MS}_j^T & m_j \end{bmatrix}$$

$$\mathbf{MS}_j^T = \begin{bmatrix} MX_j & MY_j & MZ_j \end{bmatrix}^T$$

$$\mathbf{I}_j = \begin{bmatrix} XX_j & XY_j & XZ_j \\ XY_j & YY_j & YZ_j \\ XZ_j & YZ_j & ZZ_j \end{bmatrix}$$

$$\boxed{\text{Constraints :} \quad \mathbf{J}\left(\boldsymbol{\beta}_{STD_j}\right) > \mathbf{0}}$$

Robot Constrained Identification – Constraints

Hum… well… trouble is we deal with non identifiable parameters!!
Is that theoretically/mathematically or even practically correct ??

No, of course… since such parameters have no influence on the dynamics. **Constraints involving them are futile**.

Robot Constrained Identification – Constraints

Fun fact: this problem not treated in robotics "*papers*". In short, they identify non structurally identifiable parameters.

So why dealing with such parameters ?
Is there another "*true*" (and so, well funded) theoretical framework ?
If yes, where and how ?

Robot Constrained Identification – Resolution

In Economics and Econometrics, researchers/practitioners have dealt with this topic. Proficient literature available with different types of constraints.
Overall, very "*straightforward and simple*" rationale...

First, we consider **the base parameters only** (seems obvious).
Some must be ≥ 0. We can write this as:

$$\begin{matrix} \mathbf{A}\boldsymbol{\beta} \succ \mathbf{0} \\ rank\left(\mathbf{A}\right) = p \end{matrix} \qquad \left.\begin{matrix} \mathbf{A}\boldsymbol{\beta} \succ \boldsymbol{\beta}_{LB} \\ \mathbf{A}\boldsymbol{\beta} \prec \boldsymbol{\beta}_{UB} \end{matrix}\right\} \Leftrightarrow \left\{\begin{matrix} \mathbf{A}\boldsymbol{\beta} \succ \boldsymbol{\beta}_{LB} \\ -\mathbf{A}\boldsymbol{\beta} \succ -\boldsymbol{\beta}_{UB} \end{matrix}\right.$$

LB = lower bound; UB = upper bound

Robot Constrained Identification – Resolution

Second, we write the Karush – Kuhn – Tucker (KKT) conditions and introduce the slack variables ($\mathbf{s} \geq \mathbf{0}$) to get ($\boldsymbol{\mu}$ being the Lagrangian variables $\geq \mathbf{0}$, $\mathbf{c} = \mathbf{X}^{\mathrm{T}}\mathbf{y}$ and $\mathbf{Q}_{\mathrm{XX}} = \mathbf{X}^{\mathrm{T}}\mathbf{X}$):

$$\mathbf{Q}_{XX}\hat{\boldsymbol{\beta}}_{CLS} + \mathbf{A}^{T}\hat{\boldsymbol{\mu}} = \mathbf{c}$$

$$\mathbf{A}\hat{\boldsymbol{\beta}}_{CLS} - \mathbf{s} = \mathbf{0}$$

$$\hat{s}_i\hat{\mu}_i = 0 \text{ for } i = 1 \ldots p$$ Exclusion constraint

$$\hat{s}_i, \hat{\mu}_i \geq 0 \text{ for } i = 1 \ldots p$$

Of course, $\mathbf{Q}_{\mathrm{XX}} > \mathbf{0}$ since $\mathbf{X}$ is full rank (only the base parameters considered). This eases the analysis a lot!

Robot Constrained Identification – Resolution

Third, by using the straightforward interpretation of Gourieroux and Monfort, we get:

1) The constraints are satisfied, then $\boldsymbol{\mu} = \mathbf{0}$, and

$$\mathbf{Q}_{XX}\hat{\boldsymbol{\beta}}_{CLS} = \mathbf{c}$$

follows => **usual unconstrained LS estimates**.

$$\hat{\boldsymbol{\beta}}_{CLS} = \hat{\boldsymbol{\beta}}_{LS}$$

2) The constraints are binding, then $\boldsymbol{\mu} \neq \mathbf{0}$, and

$$\mathbf{Q}_{XX}\hat{\boldsymbol{\beta}}_{CLS} + \mathbf{A}^{T}\hat{\boldsymbol{\mu}} = \mathbf{c}$$

$$\mathbf{A}\hat{\boldsymbol{\beta}}_{CLS} = \mathbf{0}$$

follows => **usual constrained LS estimates**.

Computation of covariance matrix and consistency analysis follow straight.

Robot Constrained Identification – Resolution

In practice, we can use the logarithmic barrier with the interior point method to solve the whole problem.

The problem being nonlinear, use of the **Gauss-Newton algorithm** GN).

Does it change the interpretation of Gourieroux and Monfort ?

Actually NO! We retrieve the same rationale with LS solved with the GN algorithm.
It has been shown recently that this GN algorithm can be viewed as the Generalized Method of Moments.

Still interesting to link constrained and unconstrained LS estimates.

Robot Constrained Identification – Interpretation

By considering lower and upper bounds, if the constraints are binding , we show that the constrained LS estimates are given by:

$$\hat{\boldsymbol{\beta}}_{CLS} = \begin{bmatrix} \mathbf{d} \\ \left(\mathbf{X}_U^T \mathbf{X}_U\right)^{-1} \mathbf{X}_U^T \left(\mathbf{y} - \mathbf{X}_C \mathbf{d}\right) \end{bmatrix}$$

$\mathbf{X}_u$ the n_b-p columns of $\mathbf{X}$ related to the n_b-p unconstrained parameters.

By pursuing further, we link the constrained and unconstrained LS estimates:

$$\hat{\boldsymbol{\beta}}_{CLS} = \begin{bmatrix} \hat{\boldsymbol{\beta}}_{CLS}^{C} \\ \hat{\boldsymbol{\beta}}_{CLS}^{U} \end{bmatrix} = \begin{bmatrix} \mathbf{d} \\ \hat{\boldsymbol{\beta}}_{LS}^{U} + \left(\mathbf{X}_U^T \mathbf{X}_U\right)^{-1} \mathbf{X}_U^T \mathbf{X}_C \left(\hat{\boldsymbol{\beta}}_{LS}^{C} - \mathbf{d}\right) \end{bmatrix}$$

Upper script U (resp. C) related to the n_b-p unconstrained (resp. p constrained) parameters.

Robot Constrained Identification – Interpretation

Unless $\mathbf{d} = \boldsymbol{\beta}^{C}$ (unlikely the case), the constrained LS estimates are biased. Besides, if $\mathbf{X}$ is correlated with $\boldsymbol{\varepsilon}$, the constrained LS estimates are biased.

Constraints DO NOT influence the consistency of estimates !! Though evident, this point is not treated in previous contributions. Overly restrictive constraints: LS estimates biased because the true values lie outside of the accessible region. Standard symptom: some LS estimates stick to their boundary values.

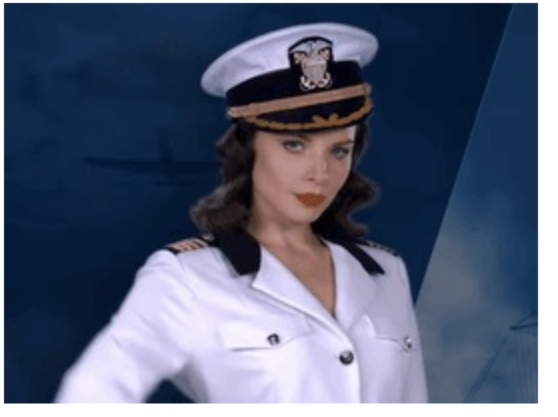
To deal with a possible correlation between $\mathbf{X}$ and $\boldsymbol{\varepsilon}$, IV is an alternative among others … but how to insert constraints into IV ?

Robot Constrained Identification – Road to IV

Tempting idea. First, inserting $\mathbf{Q}_{ZX} = \mathbf{Z}^{\mathrm{T}}\mathbf{X}$ and $\mathbf{c} = \mathbf{Z}^{\mathrm{t}}\mathbf{y}$ directly (as we would do with unconstrained IV method).
Second, compute the constrained IV "*as usual*" by writing the KKT conditions :

$$\begin{aligned} \mathbf{Q}_{ZX}\hat{\boldsymbol{\beta}}_{CIV} + \mathbf{A}^{T}\hat{\boldsymbol{\mu}} &= \mathbf{c} \\ \mathbf{A}\hat{\boldsymbol{\beta}}_{CIV} - \mathbf{s} &= \mathbf{0} \\ \hat{s}_i \hat{\mu}_i &= 0 \\ \hat{s}_i, \hat{\mu}_i &\geq 0 \end{aligned}$$

Hum… well… sounds very temping since we do "*as usual*" ($\mathbf{Z}^{\mathrm{T}}\mathbf{X}$ instead of $\mathbf{X}^{\mathrm{T}}\mathbf{X}$).



Robot Constrained Identification – Road to IV

Sounds OK if $\mathbf{Z} \sim \mathbf{X}_{nf}$ since $\mathrm{plim}(\mathbf{Z}^T\mathbf{X}) = \mathbf{X}_{nf}{}^T\mathbf{X}_{nf}/r > \mathbf{0}$ as $r \to \infty$.
Can we relax this?

Yes by considering the following IV criterion mostly employed in Econometrics:

$$f_0(\boldsymbol{\beta}) = \frac{1}{2}\|\mathbf{P}_Z\boldsymbol{\varepsilon}\|_2^2$$

with $\mathbf{P}_Z = \mathbf{Z}(\mathbf{Z}^T\mathbf{Z})^{-1}\mathbf{Z}^T$; heritage from the Two-Stage Least Squares method (2SLS).

After some calculations, we get:
$\mathbf{Q}_{ZX} = \mathbf{X}^T\mathbf{P}_Z\mathbf{X} = \mathbf{X}^T\,\mathbf{Z}(\mathbf{Z}^T\mathbf{Z})^{-1}\mathbf{Z}^T\,\mathbf{X}$ and $\mathbf{c} = \mathbf{X}^t\mathbf{P}_Z\mathbf{y}$.
Because $\mathrm{rank}(\mathbf{Z}^T\,\mathbf{X}) = n_b$ (by definition), one has $\mathbf{Q}_{ZX} > \mathbf{0}$.

Robot Constrained Identification – Road to IV

Consistency analysis does not differ from the one of the 2SLS.

We can relate the constrained and unconstrained IV estimates with:

$$\hat{\boldsymbol{\beta}}_{CIV} = \begin{bmatrix} \hat{\boldsymbol{\beta}}_{CIV}^{C} \\ \hat{\boldsymbol{\beta}}_{CIV}^{U} \end{bmatrix} = \begin{bmatrix} \mathbf{d} \\ \hat{\boldsymbol{\beta}}_{IV}^{U} + \left(\mathbf{Z}_U^T \mathbf{X}_U\right)^{-1} \mathbf{Z}_U^T \mathbf{X}_C \left(\hat{\boldsymbol{\beta}}_{IV}^{C} - \mathbf{d}\right) \end{bmatrix}$$

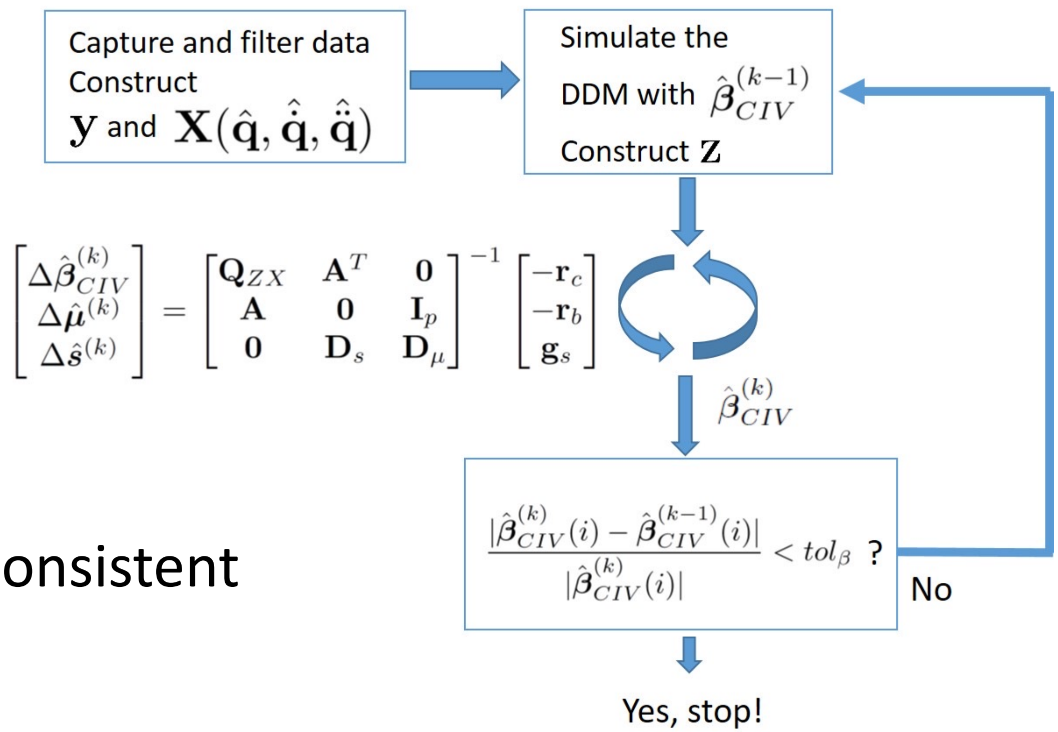
What if $\mathbf{A}\boldsymbol{\beta} > \mathbf{0}$ is replaced by $\mathbf{h}(\boldsymbol{\beta}) > \mathbf{0}$ where each $\mathrm{h}_i(\boldsymbol{\beta})$ is twice differentiable ?

In that case, $\mathbf{A}$ is the Jacobian matrix of $\mathrm{h}_i(\boldsymbol{\beta})$ around the estimates while its Hessian is $> \mathbf{0}$ (usually, $\mathbf{A}^{\mathrm{T}}\mathbf{A}$ is a good approximation).

Robot Constrained Identification – Road to IV

Note that the Hessian is required while running the GN method to solve the whole problem.

Whole algorithm of the constrained IV method, called PC-IDIM-IV method.



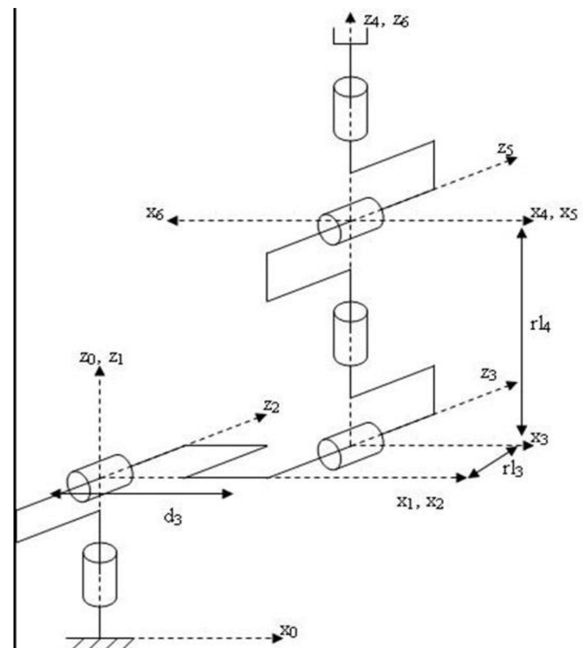
PC = Physically consistent

Experimental results – TX40 robot

Illustration of Stäubli TX40 robot

Modified Denavit and Hartenberg notation of TX40 robot

Geometric parameters defining the TX40 frames



j	$a(j)$	μ_j	σ_j	α_j	d_j	θ_j	r_j
1	0	1	0	0	0	q_1	0
2	1	1	0	$-\pi/2$	0	q_2	0
3	2	1	0	0	$d_3 = 0.225m$	q_3	$r_3 = 0.035m$
4	3	1	0	$\pi/2$	0	q_4	$r_4 = 0.225m$
5	4	1	0	$-\pi/2$	0	q_5	0
6	5	1	0	$\pi/2$	0	q_6	0

Experimental results – TX40 robot

Calculation of the base parameters

Coupling between joints 5 and 6 adds new viscous and Coulomb friction parameters Fvm_6 and Fcm_6.

From 78 standard parameters, 60 base parameters.

SYMORO+ software used to automatically calculate the customized symbolic expressions of the IDM.

	1	2	3	4	5	6
XX_j	0	XX_{2R}	XX_{3R}	XX_{4R}	XX_{5R}	XX_{6R}
XY_j	0	XY_2	XY_3	XY_4	XY_5	XY_6
XZ_j	0	XZ_{2R}	XZ_3	XZ_4	XZ_5	XZ_6
YY_j	0	x	x	x	x	x
YZ_j	0	YZ_{2R}	YZ_3	YZ_4	YZ_5	YZ_6
ZZ_j	ZZ_{1R}	ZZ_{2R}	ZZ_{3R}	ZZ_{4R}	ZZ_{5R}	ZZ_{6R}
MX_j	0	MX_{2R}	MX_3	MX_4	MX_4	MX_6
MY_j	0	MY_2	MY_{3R}	MY_{4R}	MY_{5R}	MY_{6R}
MZ_j	0	x	x	x	x	x
m_j	0	x	x	x	x	x
Fv_j	Fv_1	Fv_2	Fv_3	Fv_4	Fv_5	Fv_6
Fc_j	Fc_1	Fc_2	Fc_3	Fc_4	Fc_5	Fc_6
Ia_j	0	0	Ia_3	Ia_4	Ia_5	Ia_6

Experimental results – Data filtering

Data filtering, maximum bandwidth for the sixth joint, $\omega_{dyn} = 10\text{Hz}$:

- **q** filterted with a 50 Hz fourth-order Butterworth filter ($\omega_{fb} \geq 5\, \omega_{dyn}$);

- $\hat{\dot{\mathbf{q}}}, \hat{\ddot{\mathbf{q}}}$ calculated with central differentiation of **q**;

- Parallel decimation carried out with a lowpass Tchebyshef filter with a cutoff frequency of 20 Hz ($\omega_{fp} \geq 2\, \omega_{dyn}$).

Filters tuned accordingly to rules given by Maxime Gautier.

Experimental results – Actual data

From 60 base parameters, only 27 are significant => set of essential parameters; validated with a F-test.

PC-IDIM-IV converges in 40s, very good (not time consuming).

Relative error of 5%-7% (direct and cross validations)

With appropriate data filtering, $\mathbf{X}$ close to $\mathbf{X}_{\mathrm{nf}}$ (noise-free observation matrix).

Relative errors - Direct validations

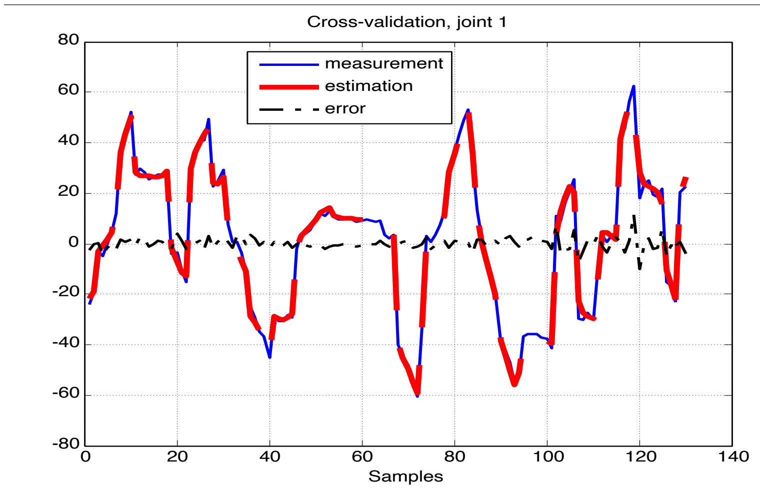
Joint j	IDIM-LS	IDIM-IV
1	5.2%	5.4%
2	5.0%	5.3%
3	5.0%	5.1%
4	5.6%	5.7%
5	7.2%	7.2%
6	7.1%	7.5%
Joint j	PC-IDIM-LS	PC-IDIM-IV
1	5.2%	5.4%
2	5.0%	5.3%
3	5.0%	5.1%
4	5.6%	5.7%
5	7.2%	7.2%
6	7.1%	7.5%

Relative errors -Cross-validations

Traj.	IDIM-LS	IDIM-IV
1	6.1%	6.4%
2	6.3%	6.8%
3	6.7%	7.0%
Traj.	PC-IDIM-LS	PC-IDIM-IV
1	6.1%	6.4%
2	6.3%	6.8%
3	6.7%	7.0%

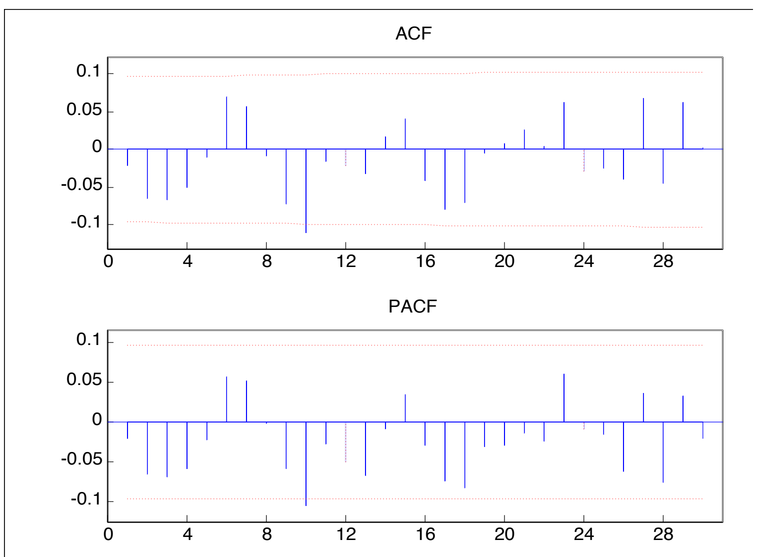
LS estimates close to IV estimates. Using constraints and/or (un)constrained IV method does not improve LS estimation. Constraints do not improve results.

Experimental results – Actual data

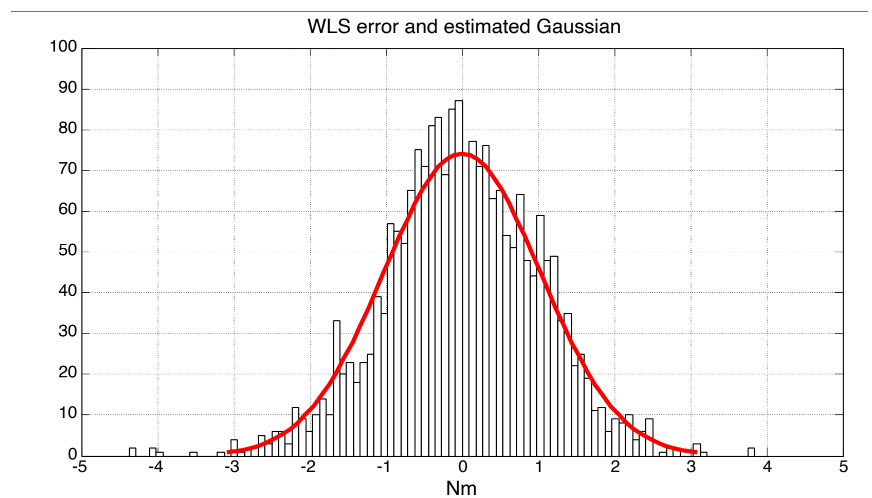


Cross test validation: the reconstructed torque matches the measured one (first joint); relative error of 6% (compatible with 5%)

Plot provided by the ACF function shows that the residuals are white.



The distribution of the residuals looks like Gaussian.



Experimental results – Downgraded data

Data downgraded with same data filtering.

Relative error of 5%-8% (direct and cross validations) with IV while 25%-30% with LS. LS estimates biased despite data filtering. Persistent bias due to the correlation between $\mathbf{X}$ and $\boldsymbol{\varepsilon}$.

Evident that constraints do not help, no clear improvements (if any).

PC-IDIM-IV converges in less than 1 minute, still very good!!

Relative errors - Direct validations

Joint j	IDIM-LS	IDIM-IV
1	25.3%	6.1%
2	24.7%	5.9%
3	25.2%	6.0%
4	25.8%	5.9%
5	27.5%	8.0%
6	28.0%	8.1%
Joint j	**PC-IDIM-LS**	**PC-IDIM-IV**
1	25.3%	6.1%
2	24.7%	5.9%
3	25.2%	6.0%
4	25.8%	5.9%
5	27.5%	8.0%
6	28.0%	8.1%

Relative errors -Cross-validations

Traj.	IDIM-LS	IDIM-IV
1	26.4%	7.1%
2	26.5%	7.6%
3	27.8%	7.9%
Traj.	**PC-IDIM-LS**	**PC-IDIM-IV**
1	26.4%	7.1%
2	26.5%	7.6%
3	27.8%	7.9%

LS estimates mismatch IV estimates. Using constraints do not prevent LS estimation from bias/inconsistency; only (un)constrained IV methods do.

Experimental results – Error in controllers

Controllers rarely perfectly known (industrials reluctant to share their know-how).

We have to identify them. Today it is not a big deal since an ARMA model is generally enough.

Toolboxes available => we identify the controllers and then run (un)constrained IV methods.

Relative errors - Direct validations

Joint j	IDIM-IV	PC-IDIM-IV
1	6.1%	5.9%
2	6.3%	5.8%
3	5.9%	5.5%
4	6.4%	6.2%
5	7.9%	7.6%
6	8.1%	7.9%

Relative errors -Cross-validations

IDIM-IV	PC-IDIM-IV
7.1%	6.8%
7.7%	7.2%
8.1%	7.5%

IV methods perform very well: same results as those obtained with the knowledge of controllers. Convergence in less than 1 minute!!
Tests with downgraded bandwidth: IV methods OK until $\omega_{dyn}/3$.

Good robustness of IV methods against errors in controllers.

That said, constraints do not improve the estimation significantly…

Conclusion – General

Using constraints could be helpful for system identification though we introduce *a priori* knowledge.

Make sure that constraints use only the identifiable parameters.
Make also sure that they are not overly restrictive.

Constraints do not prevent from bias of LS estimation, only IV methods do among others.

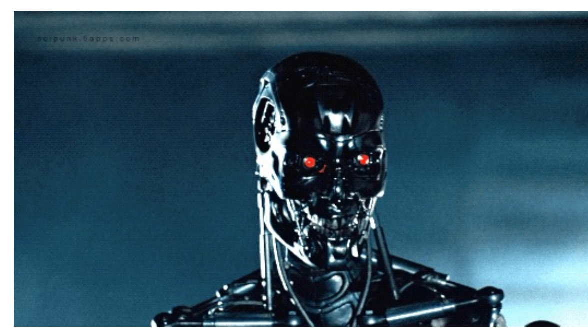
Theory from Econometrics is very solid mathematically speaking.
Of course, if you feel brave, feel free to read books of "*pure*" mathematics ☺

Conclusion – Robots

The PC-IDIM-IV converges very fast: 60 parameters identified in less than 1 minute.
Good performance!

However, for robot identification, the use of constraints does not improve the estimation significantly.
Why ? Consistency => constraints are satisfied.

That said, it does not mean that constrained identification is useless for robots.
Perhaps for humanoids identification…



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Thank you for your

attention !